

# Better mathematics conference

Subject leadership

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# Aims of workshops

To strengthen your leadership role in improving:

- the quality of mathematics teaching
- the subject-specific focus and impact of monitoring activities, particularly work scrutiny.

# Content of workshops



- Points for you to think about individually
- Activities for you to do in pairs

The workshop information pack contains the materials you will need for the activities.

During the session, please do not hesitate to speak about the activities to supporting HMI. If you have further questions, please note them down on paper and hand to HMI.

# Workshop sections



- Improving the quality of teaching by:
  - increasing problem solving, reasoning and conceptual understanding
  - reducing variability
  - identifying and overcoming misconceptions
  
- Improving the subject-specific focus and impact of monitoring activities, particularly work scrutiny

Increasing problem solving, reasoning and  
conceptual understanding

# Deepening a problem



Many pupils spend too long working on straightforward questions.

Such questions can often be changed into problems that develop pupils' thinking and reasoning skills.

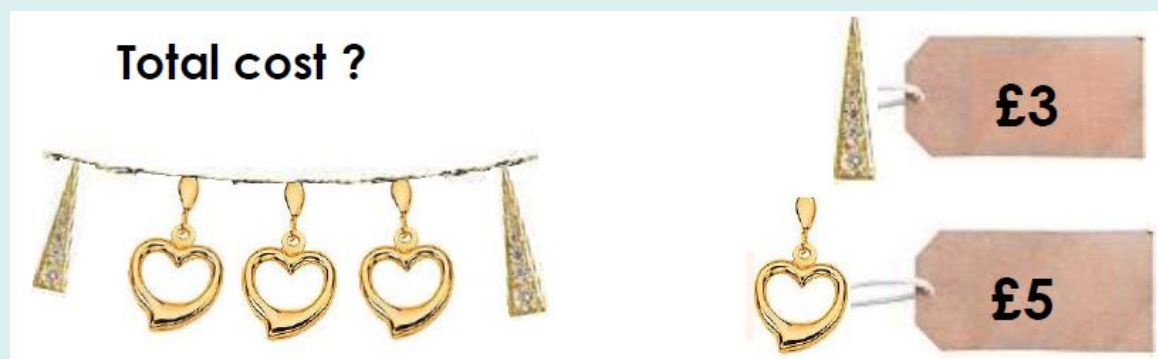
The following activities may help you to:

- stimulate discussion in the school about problem solving and reasoning
- support your colleagues to reflect on the questions they use.

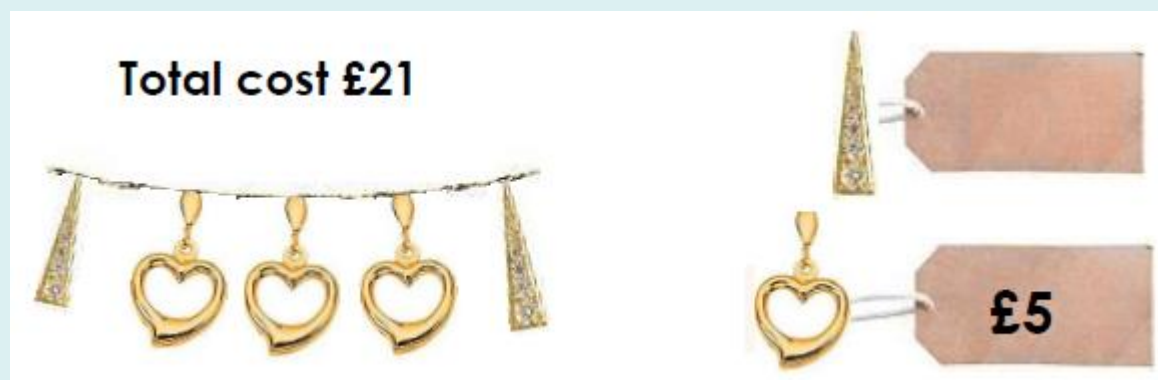
# Deepening a problem

- With your partner, decide which version involves more problem solving and reasoning, and why.

Version 1



Version 2



# Deepening a problem



Too often, teachers provide a set of questions like version 1, with only the numbers (costs and quantities of each shape) changed each time.

While this provides practice with multiplication, it does not develop pupils' problem-solving skills. After the first question, pupils do not have to think how to interpret each one or decide on the steps to take to solve it.

Pupils may also form the false impression that mathematics is repetitive and does not require deep thought.



# Deepening a problem



Version 2 requires the use of deeper problem-solving skills.

Pupils have to decide where to start. They need to think how the total was obtained and work backwards to find the missing cost. Choosing the correct order of steps is crucial.

Reversing the problem makes it more complex, in this case from

- given costs of individual items, find cost of necklace (do hearts and triangles in either order, then add) to
- given cost of necklace, find cost of an individual item (do hearts, subtract from total, then halve; so an efficient order of operations is multiply, subtract, then divide).

# Deepening a problem



You can ask more open questions using the same context.

One way of doing this is to give pupils a total cost and ask them for all the possible different combinations of hearts costing £5 and triangles costing £3.

For example, a necklace costing £20 could have either 4 hearts or 1 heart and 5 triangles.

Such open questions enable pupils to develop problem-solving skills as well as improving fluency with multiplication tables and calculation.

# Money problem



With your partner, decide which question involves more problem solving and reasoning, and why.

## Question 1

Jeans cost £13.95. They are reduced by  $\frac{1}{3}$  in a sale.  
What is their price in the sale?

Dan buys the jeans. He pays with a £10 note.  
How much change does he get?

## Question 2

Jeans cost £13.95. They are reduced by  $\frac{1}{3}$  in a sale.  
Dan buys the jeans. He pays with a £10 note.  
How much change does he get?

# Money problem



- Q1 has two one-step parts, one on calculating the fraction of an amount and one on subtraction.
- Q2 is a two-step question so involves more problem solving. It requires pupils to recognise that the sale price needs to be found first, then used with subtraction.
- The following version of the question develops problem-solving skills more deeply than Q2 as the pupil has to decide on the approach and what to calculate. It allows an elegant approach using a sense of number, without an exact calculation. It also asks explicitly for reasoning.

Jeans cost £13.95. They are reduced by  $\frac{1}{3}$  in a sale.

Dan has £10. Does he have enough money to buy the jeans?  
Explain why.

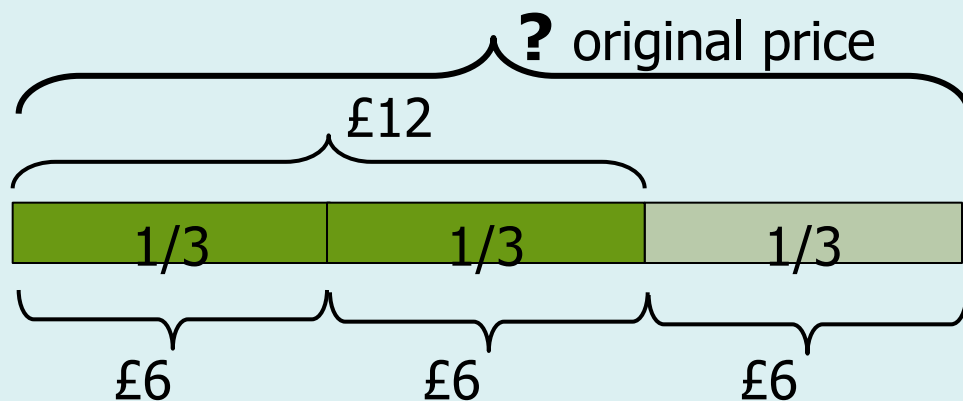
# Money problem

## Question 4

A different pair of jeans is also reduced by  $\frac{1}{3}$  in a sale.  
The sale price is £12.

What was the original price?

- Q4 is venturing into reverse calculations – a more difficult idea, but a natural next step. The easier sale price of £12 allows the concept to be explored.
- Pupils need to work backwards,  $£12 \div \frac{2}{3}$ . A common error is to find  $\frac{1}{3}$  of £12 = £4, then adding to get £16.



The bar model is a useful representation for such problems (& reverse %)

# Ways of deepening a problem

Problems can be adapted by:

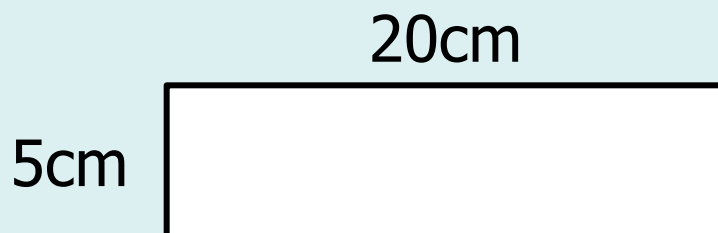
- removing intermediate steps
- reversing the problem
- making the problem more open
- asking for all possible solutions
- asking why, so that pupils reason
- asking directly about a mathematical relationship.

Remember, you can:

- improve routine and repetitive questions by adapting them
- set a rich problem or investigation instead
- discuss alternative approaches to solving the problem
- set problems that go more deeply into the topic.

# Deepening an area problem

- What is the area of this rectangle?



- With your partner, adapt this question to encourage pupils to think harder about how to solve it, and better develop their problem-solving and reasoning skills and/or conceptual understanding of area of rectangle.
- The problem you devise should be based on the same 20cm by 5cm rectangle.

# Some problems based on 5x20 rectangle



- Which square has the same area as this rectangle?
- Find all the rectangles with whole-number side lengths that have the same area as this one.
- How many rectangles have an area of  $100\text{cm}^2$ ? Explain.
- If I halve the length and double the width, what happens to the area? What if I double the length and halve the width?
- Imagine doubling the length and width of the rectangle (do not draw it). Think: what will the area of the new rectangle be? Now draw it and check its area. Explain your findings.
- What happens to the area and the perimeter when you cut a piece from the corner of the rectangle? Is it possible for the perimeter to be the same or larger than originally? How?



Think for a moment ...

How could you support your colleagues  
in deepening problems for the next topic  
they will be teaching?



# Reducing variation in the quality of teaching

Think for a moment ...

How is the formula for area of a rectangle taught in your school?



# Approaches to rectangle area



A school was strengthening its mathematics scheme of work by providing guidance on approaches that help to develop pupils' understanding and problem-solving skills.

As part of this work, the mathematics subject leader asked staff to bring to a meeting examples of how they introduced the formula for the area of a rectangle when they last taught it.

# Approaches to rectangle area

At the meeting, five staff explained how they introduce the formula for area of a rectangle.

Look in your pack at what they said.

Their approaches include a number of strengths and weaknesses.

With your partner, identify:

- strengths in the way pupils' understanding of area was developed
- weaknesses in conceptual development in some of the approaches.

# Approaches to rectangle area

Teachers C and E used approaches that develop conceptual understanding particularly strongly.

## Teacher C

- ensured pupils saw how repeated addition of rows or columns led to the formula, using images
- helped pupils to apply the commutativity of multiplication (for example, that  $3 \times 4 = 4 \times 3$ ) to the formula for area, so recognise that length  $\times$  width = width  $\times$  length, and see that it did not matter which dimension was written first or was longer. This encouraged pupils to be flexible in visualising rectangles as made up of rows or columns.

# Approaches to rectangle area

## Teacher E

- ensured pupils saw how rows were combined or columns combined to give the same area, regardless of orientation, so realised the product was commutative ( $L \times W = W \times L$ )
- supported development of pupils' flexibility in reasoning and use of visual images, including through working backwards from area to find length and width (finding pairs of factors)
- ensured pupils became aware of the formula for finding area through its structure
- also developed pupils' investigatory and problem-solving skills, such as predicting, checking and explaining whether all solutions had been found.

# Approaches to rectangle area



Approaches described by staff A, B and D contained some weaknesses in development of conceptual understanding.

## Teacher A

- gave no conceptual introduction, just stated the formula
- stressed that L is the larger dimension. This does not help pupils realise L and W can be multiplied in either order or apply the formula to squares (special case of rectangles)
- provided practice at mental multiplication and measuring lengths. This appears to vary the activities and link to skills in other aspects of mathematics but does not help pupils' understanding of area or development of reasoning
- used harder numbers, rather than more complex area concepts, for abler pupils.



# Approaches to rectangle area

## Teaching assistant B

- used an everyday context but, as pool length is longer than width, it may reinforce the idea that  $L$  is longer than  $W$ .

## Teacher D

- A common experimental approach, often not used well enough to help pupils understand where the formula comes from. Results from counting squares are linked to  $L \times W$  just by looking at the table to spot connections, but without the structural step of repeated addition of rows or columns then reasoning why this simplifies to  $\text{area} = L \times W$  or  $W \times L$ .
- Also, errors in counting squares or finding lengths can cause a mismatch between pupils' results and the formula  $L \times W$ , which then appears to pupils to work only sometimes.

# Impact of different approaches



Pupils of teachers C and E have flexible visual images of how the area of a rectangle is built up. They can return to these images in future, so are less likely to forget the formula or use an incorrect one, as they can reproduce it from first principles.

- This is what mathematics teaching should be aiming for.

When pupils of teachers A and D need to work out the area of a rectangle some time in the future, they are reliant on memory and likely to confuse the formulae for area and perimeter (often taught at the same time). They have no visual images to return to, to help them find their **own** way out of a quandary.

- Mathematics teaching needs to equip pupils with the confidence and expertise to find their own way forward whenever they are unsure what to do.

# Approaches to rectangle area



The school recognised that, without guidance, pupils' experience would be dependent upon which teachers were teaching area of a rectangle next year. It wanted to tackle any gaps in understanding pupils had developed when area of a rectangle was introduced in primary school.

If the weaker approaches were to be used, pupils would:

- not receive a conceptual explanation and therefore not fully understand rectangle area
- become too reliant on remembering the formula rather than working it out for themselves
- not be well enough prepared for future work on area, such as on parallelograms, triangles or compound shapes.

# Approaches to rectangle area



The school decided to draw upon its teachers' good practice and provided this guidance:

- choose the introduction and the conceptual development of the formula for area of a rectangle used by either teacher C or teacher E
- from the outset, ensure questions are not repetitive and use problems to deepen understanding and reasoning about area
- present questions and problems in different ways e.g. in reverse so length is found given area and width, in words requiring rectangles to be visualised or drawn, as composite shapes, in contexts, requiring comparison without calculation, as investigations and asking for explanations.

# Approaches to other topics



You might find the following general questions useful in discussions with your colleagues about the teaching of other topics, perhaps identified through monitoring of teaching or question level analysis of test results.

1. How well does your introduction develop conceptual understanding?
2. How repetitive are your questions? How soon do you use questions that reflect the breadth and depth of the topic?
3. At what stage do you set problems? How well do they deepen understanding and reasoning?
4. Are questions and problems presented in different ways?

Think for a moment ...

Identify a topic you would find it helpful to discuss in this way with your colleagues.

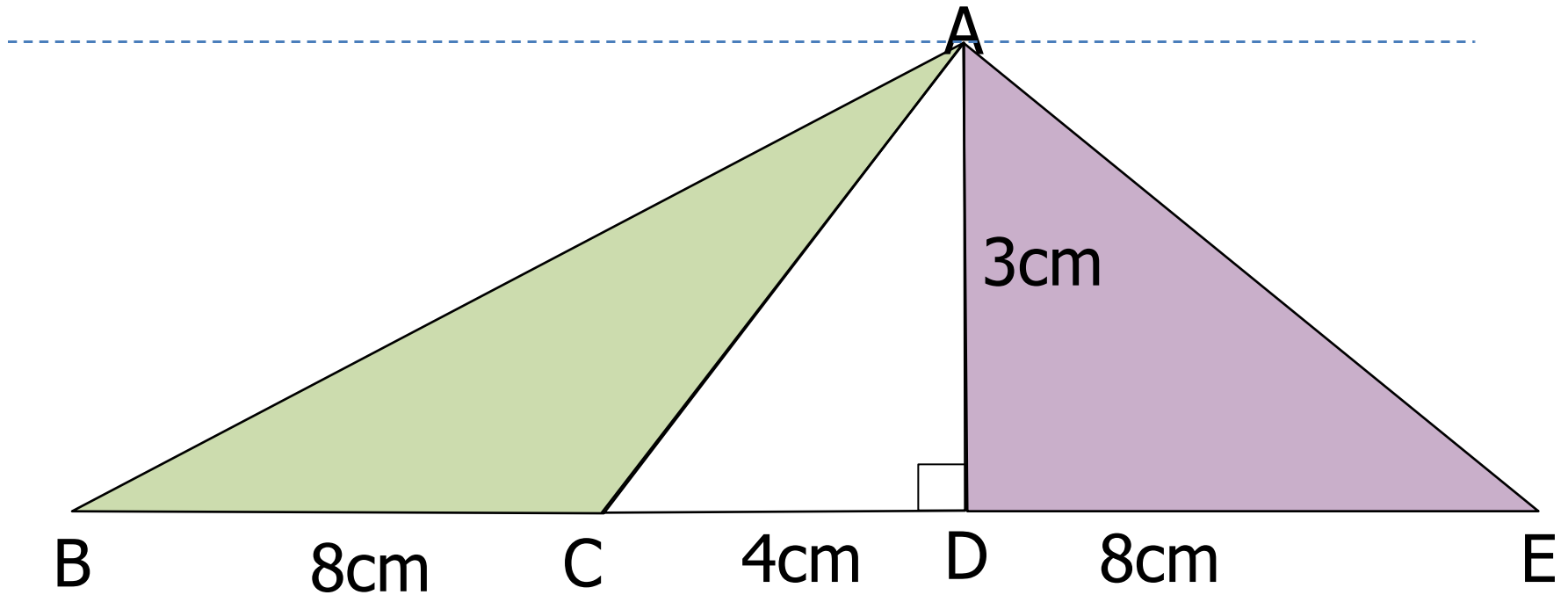


# A consistent conceptual approach



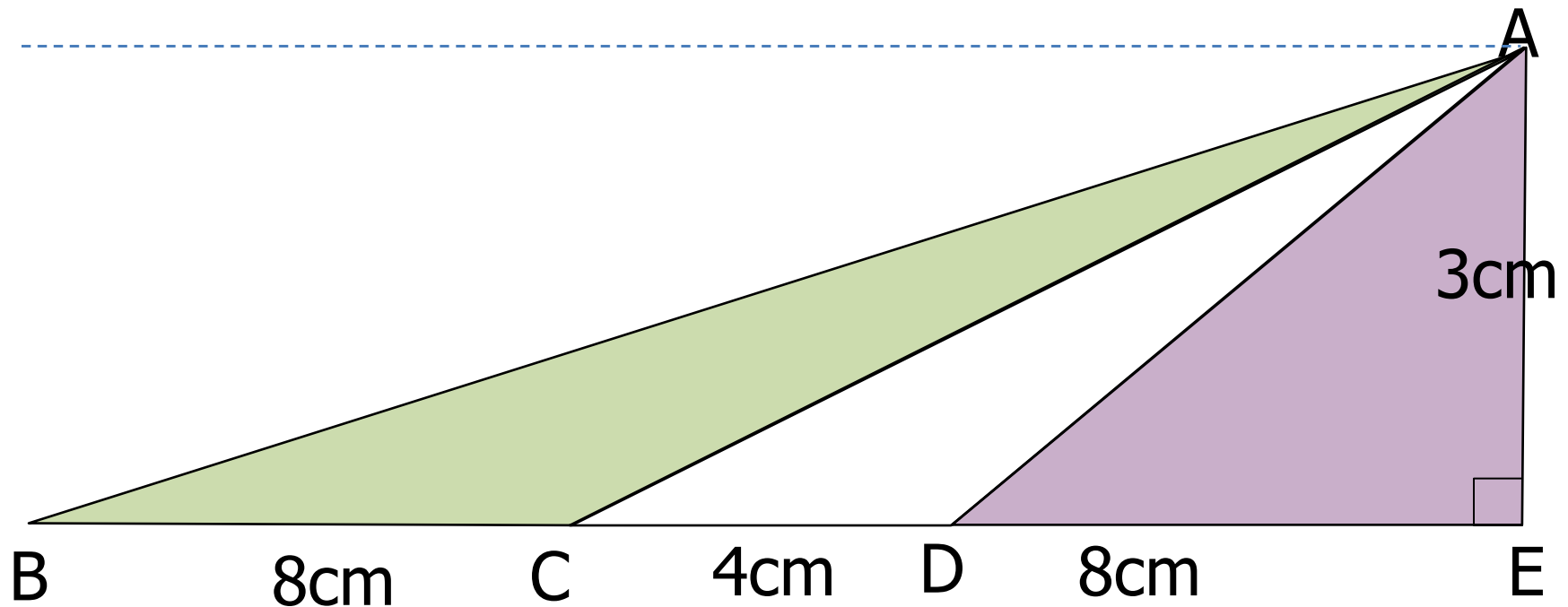
When concepts are linked, such as area of rectangles, parallelograms and triangles, pupils' understanding is helped if these are developed in a coherent way across the school. Consistent use of practical equipment and visual images plays an important part in this.

ICT can help pupils visualise the effect on area of changes in a shape's lengths or angles, and reason why.

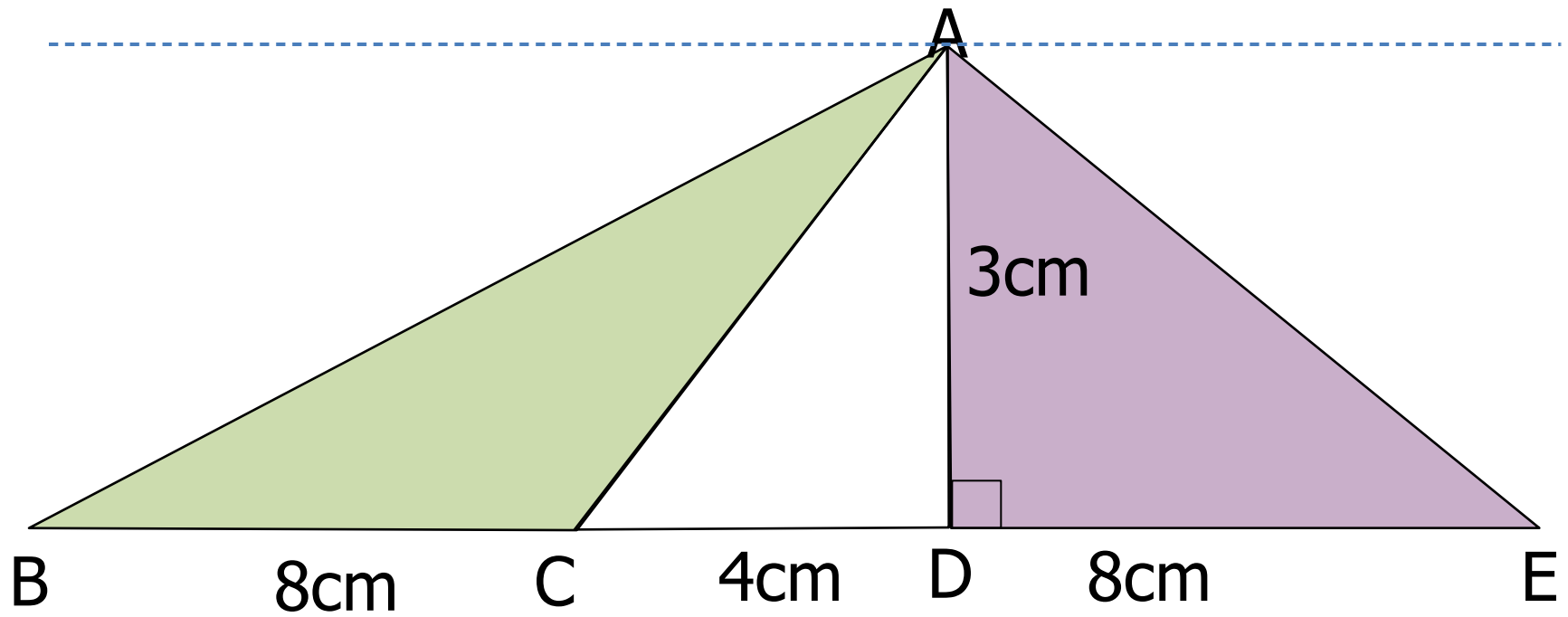


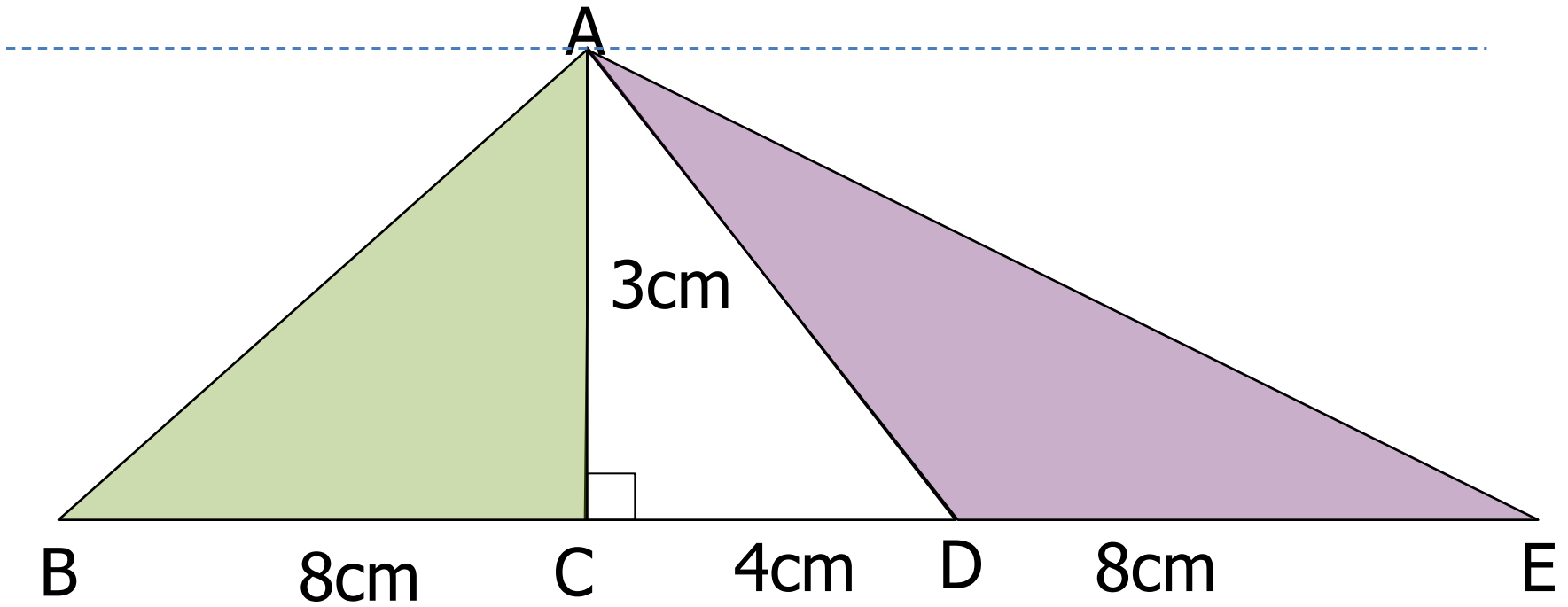
What is the area of each of these three triangles?

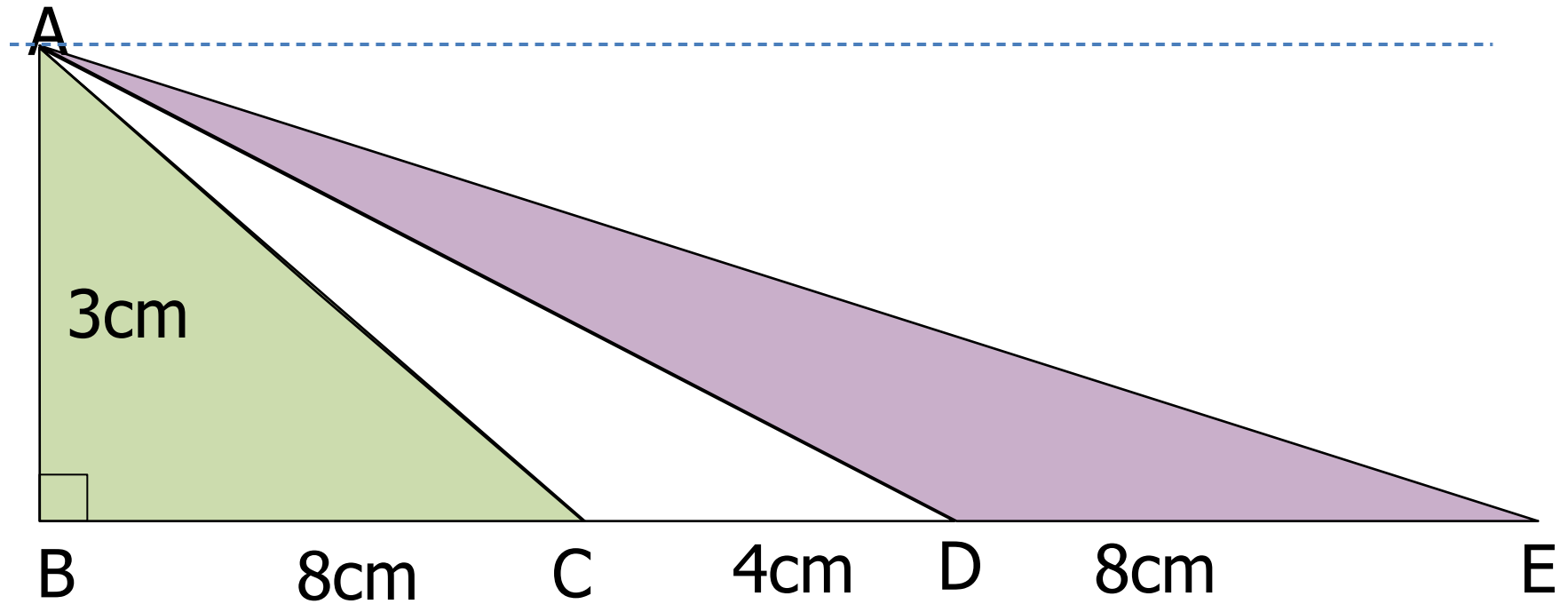


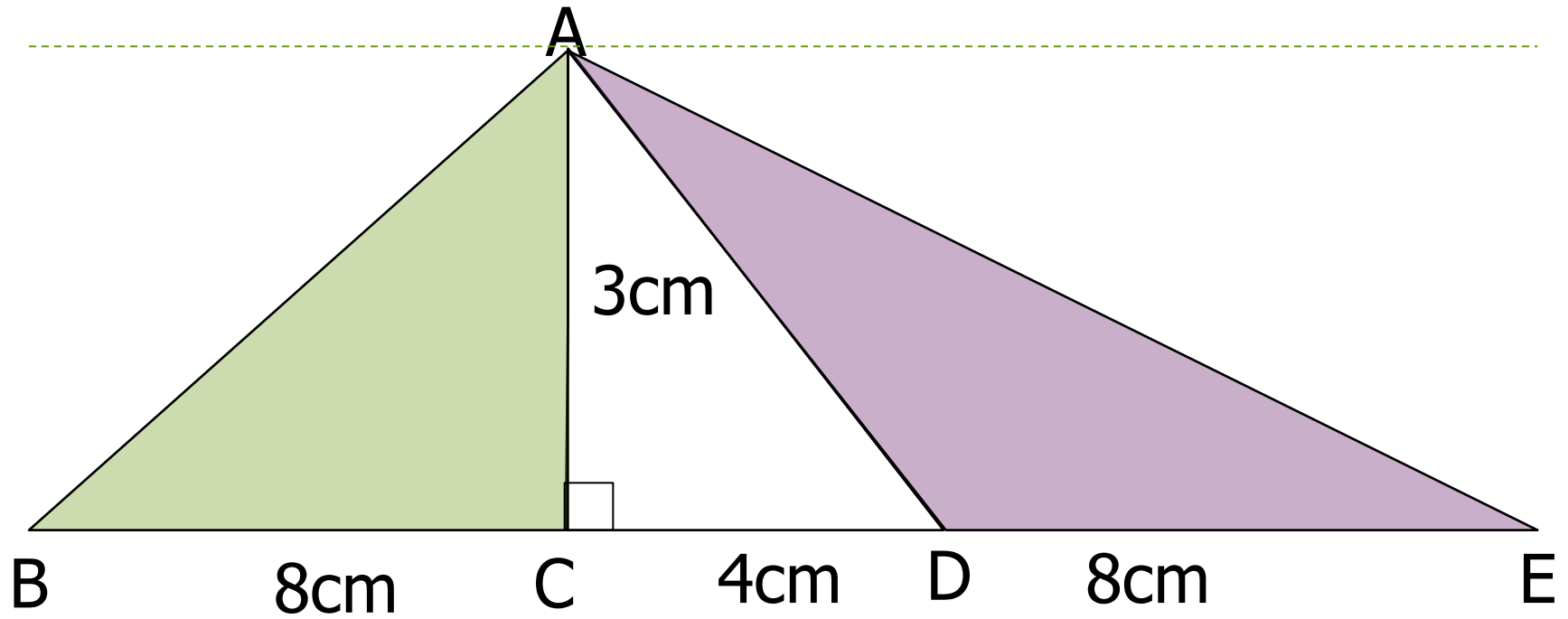


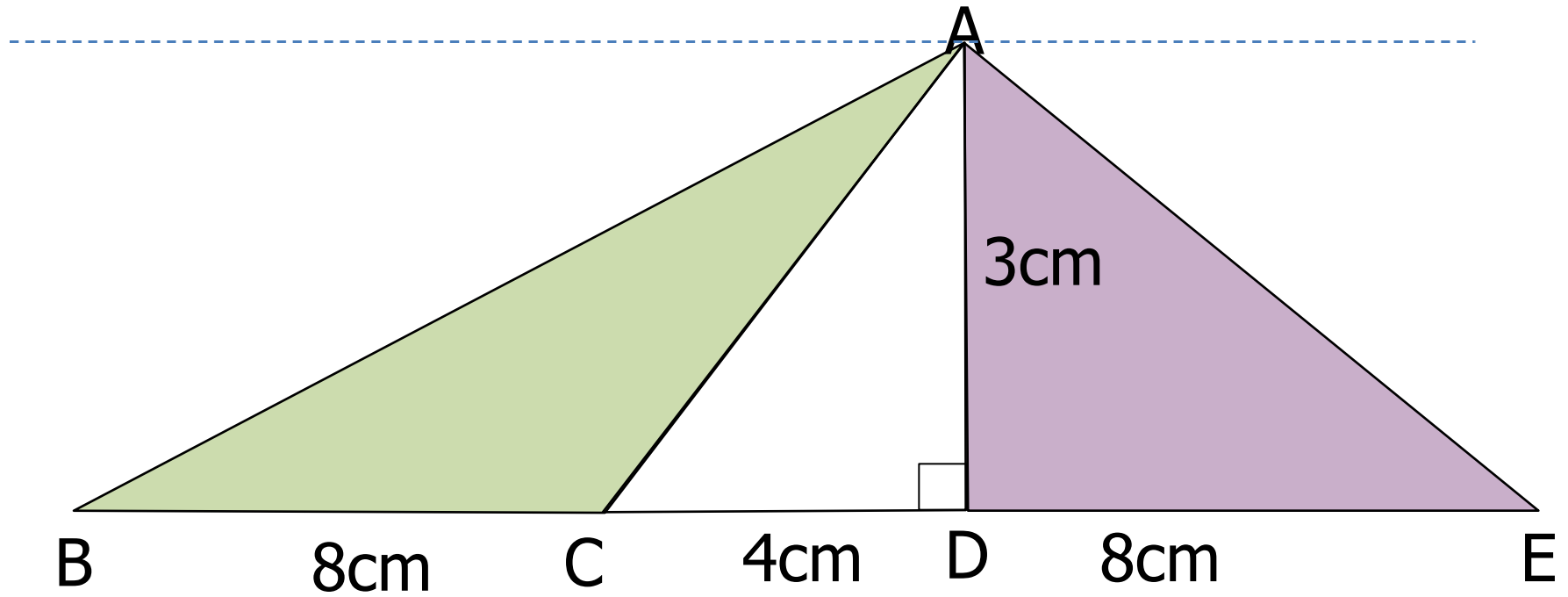
Now, what is the area of each of the triangles?  
Why?





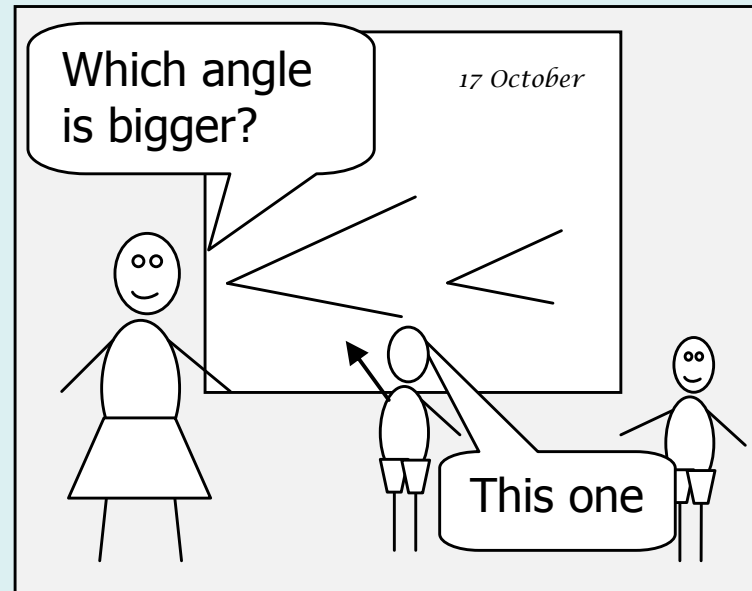






# Identifying and overcoming misconceptions

# What is the boy's misconception?



- Pupils often see the angle in terms of the length of the lines or the space between them, rather than the rotation from one arm to the other.



# Misconceptions



- In your pack is a table of several errors caused by:
  - underlying misconceptions
  - unhelpful rules
  - lack of precision with the order of language and symbols.
- With your partner, see if you can identify the underlying misconception or cause of each error.

# Underlying misconception

Take 6 away from 11

$$6 - 11 = 5$$

Writing a subtraction in the order the numbers occur and ignoring some words e.g. 'from' – so it is the wrong way round

$$6 \div \frac{1}{2} = 3$$

Applying the unhelpful rule 'when you divide, the answer is smaller', often introduced when starting work on division

*In order, smallest first:*

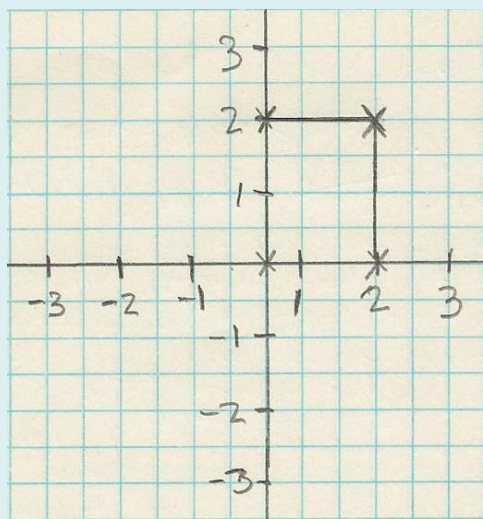
3.2    3.6    3.15    3.82    3.140

Reading the decimal part as a whole number (like money) e.g. three point fifteen, rather than using place value

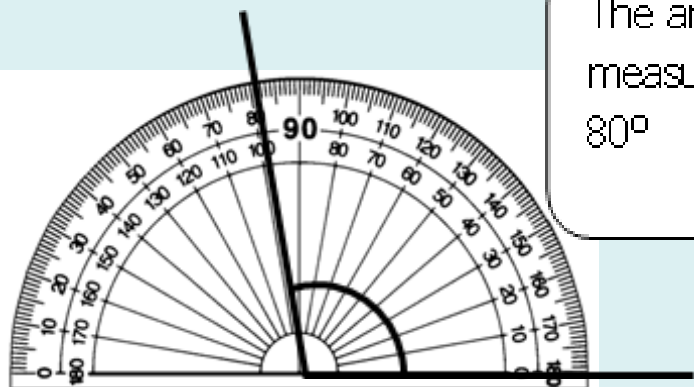
# Underlying misconception

$2.7 \times 10 = 2.70$	Using the unhelpful rule 'to multiply by 10, add a zero', rather than place value.
$32.48 = 32.5$ to 1dp, $= 33$ to the nearest whole number	Successive rounding, first to 1dp and then rounding that figure up to the next whole number, instead of rounding 32.48 directly to the nearest whole number, 32. (The answers 32.00 and 32.0 are also incorrect.)
$10\% \text{ of } 70 = 70 \div 10 = 7$ $20\% \text{ of } 60 = 60 \div 20 = 3$	$10\% = \frac{10}{100} = \frac{1}{10}$ . Omitting the middle step to say 'for 10%, $\div$ by 10' leads incorrectly to 'for 20%, $\div$ by 20'

# Underlying misconception



Not realising that numbers must be equally spaced on axes, in particular that the distance from the origin to 1 is the same as other intervals.



The angle  
measures  
80°

Reading the wrong scale on a protractor, often the outside scale – not starting at 0 on one arm and counting through the rotation between the arms.

# Misconceptions

- These misconceptions, some of them arising from incorrectly applied poorly worded rules, inhibit pupils' understanding of the topic itself and all future work that depends upon it.
- For example, using unequal intervals on an axis can impede learning about shapes or graphs that are then plotted.

Think for a moment ...

*Take 6 away from 11*

$$6 - 11 = 5$$

What future learning might be impeded?



- Incorrect or uncertain ordering of subtractions can inhibit understanding of how to combine negative numbers and algebraic terms, and lead to incorrect column subtraction. A weak use of sequencing in mathematical language from the outset in the Early Years can contribute to this.
- Teachers should model the correct ordering of mathematical language to express operations and comparisons, and check that pupils do this orally and in symbols, for example:
  - 'the pen is shorter **than** the pencil, so the pencil is longer **than** the pen'
  - $3 < 8$ , so  $8 > 3$  (reading both ways in words & symbols).

Think for a moment ...

How might you find out about  
misconceptions across the school?





# Finding out about misconceptions



You might spot them in your monitoring role (as when teaching):

- in pupils' written work (during lessons or when scrutinising their work or assessments)
- when circulating and listening to pupils during lessons.

You might check deliberately for them when talking to pupils about mathematics:

- in lessons during learning walks/observations
- to seek their views and probe their understanding (perhaps through selecting a group termly).

If a pupil's earlier work shows a misconception, you might check whether the pupil has now overcome it.

Think for a moment ...

How might you help colleagues use  
misconceptions well in their teaching?



# Ways to help colleagues



- Help staff to be aware of misconceptions that:
  - pupils may bring to the lesson
  - might arise in what is being taught.
- Encourage staff, when planning a topic, to discuss mistakes that pupils commonly make in that topic and explore the misconceptions that underpin them. Also help staff to:
  - plan lessons to take account of the misconceptions
  - look out for misconceptions by circulating in lessons.

Bear in mind, it is more effective to address misconceptions directly than to avoid or describe them. Giving pupils carefully chosen examples to think about deeply allows pupils to reason for themselves why something must be incorrect.

- Misconceptions built up early in primary school, or later, can substantially impede much of a pupil's future mathematical learning. They are evident in what pupils write in their books and do in lessons.
- Not all errors come from misconceptions – they might be just small slips or inaccuracies. It is important that teachers distinguish between misconceptions and slips during lessons and when marking in order to help pupils overcome them.
- It may take longer than one lesson to help a pupil overcome a substantial misconception. Support is best timed to take place before teaching new work that depends on it because pupils cannot understand such work without overcoming the misconception. Often, written comments alone will be insufficient to achieve this.

# Work scrutiny

# The potential of work scrutiny



To check and improve:

- teaching approaches, including development of conceptual understanding and reasoning
- depth and breadth of work set and tackled, including levels of challenge
- problem solving
- pupils' understanding and misconceptions
- assessment and its impact on understanding.

To look back over time and across year groups at:

- progression through concepts for pupils of different abilities
- how well pupils have overcome any earlier misconceptions
- balance and depth of coverage of the scheme of work, including reasoning and problem solving.

- Look at some/all of the samples of work from pupils in Years 6, 4, 5 and 2. The school has had a focus on increasing problem solving.
- For each piece of work, consider the:
  - teaching approach, including development of conceptual understanding and reasoning
  - depth and breadth of the work set and tackled, including levels of challenge
  - quality of problem solving
- Identify any strengths and weaknesses for each individually and then across the four.

# Work scrutiny – pupil A

- Teacher has used a suitable practical approach of rearranging cards, which helps pupils keep the sign with its term.
- Breadth and challenge from the outset. Only Q1 is an 'easy case', others mix + and – in the questions and the answers.
- Q6 & Q7 make pupils think hard. Q6 has missing information that pupils need to deduce before finding the perimeter and Q7 asks for a proof.

**A** Collecting like terms worksheet

Remember what we learned about this when we moved the cards around:

$5x$	$-4y$	$-3x$	$+3y$
$(+)5x$	$-4y$	$-3x$	$+3y$
$(+)5x$	$-3x$	$-4y$	$+3y$
$= 5x$	$-3x$	$-4y$	$+3y$
$= (5-3)x + (-4+3)y = 2x - y$			

- $2x + 3y + 3x + 4y =$   $5x + 7y$  ✓
- $-3u + 5u + 2v - 2v =$   $2u + 0v$  ✓  $= 2u$
- $11x + 3 - 7x - 5 =$   $4x - 2$  ✓
- $p + 3q - 2p - q =$   $-p + 2q$  ✓
- $3m - 2n - m - 3n =$   $2m - 5n$  ✓

6. Find the perimeter of this shape:

*I think you forgot the unmarked sides*  
 $8w + 3x$  ✗  
 Correction please!  
 $= 10w + 6x$  ✓

7. Prove that if you add any three consecutive numbers together, the answer is always divisible by 3. [Hint: Let the first number be  $n$ , so the next two are ...]

$n + n+1 + n+2 = 3n+3$  ✓  
 $(3n+3) \div 3 = n+1$  ✓ good  
*How does this prove it can be divided by 3?*  
*I think you've got the idea. ssa 17/6*  
 Try this:  $4(x+y) + 2(x-y) =$  ✓  
 $4x + 4y + 2x - 2y = 6x + 2y$  ✓  
 Excellent. ssa 24/10



# Work scrutiny – pupil B

- Teacher's comment suggests the taught method is to 'divide by the denominator and times by the numerator'. Need to talk to pupil to establish if he understands why the rule works.
- As written by pupil B, the problems seem to follow the same format. However, his calculations are largely correct, e.g.  $\frac{3}{5}$  of 2kg = 1.2kg;  $\frac{1}{3}$  of 4hr = 1.333...hr = 1hr 20min suggesting he knows how to calculate fractions of quantities.
- The problems might be varied – need to check with teacher. Some may have two steps, e.g. Q4 might be to reduce 650 by  $\frac{7}{10}$ , so  $650 - 455 = £195$ , not £205 as pupil has written.

Remember, divide by the denominator  
Times by the numerator

Q1: To find fractions of quantities. (not sure on no)

①  $\frac{3}{5}$  of 2kg = 4.5kg. ~~1.2kg~~ 1.2kg

②  $\frac{1}{3}$  of 4 hours = 1.33 hours. ✗ ✗

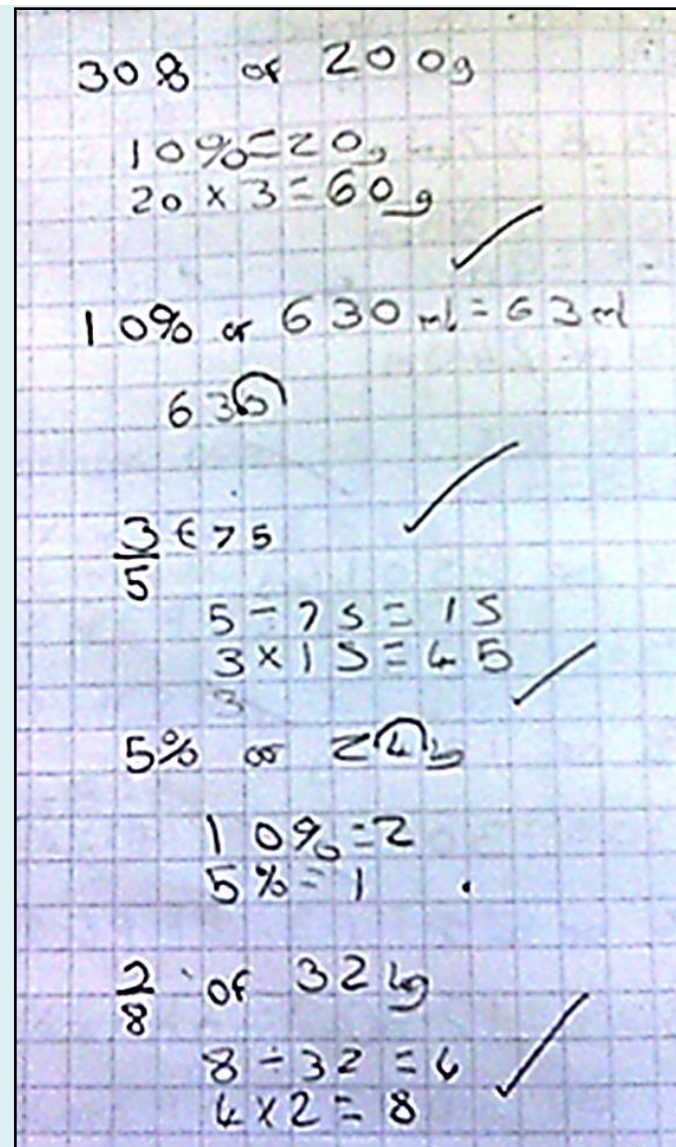
③  $\frac{3}{8}$  of 12kg = 4.5kg. ✗ ✓

④  $\frac{7}{10}$  of 650 = ~~£455~~ £205. ✗ ✗

⑤  $\frac{1}{5}$  of 455 = £91. ✗ ✗

# Work scrutiny – pupil C

- Pupil C uses one method for calculating fractions of quantities and another for percentages.
- Unclear whether she has been taught the conceptual link between percentages and fractions. (How would she calculate 25%?)
- Pupil knows that dividing by 10 gives 10%, but struggles when quantity is not a multiple of 10. She appears to move the decimal point by one place – has she been taught to do this?
- Problems appear to be very similar with no increase in complexity.



Handwritten work on grid paper showing calculations for fractions and percentages:

- $308 \text{ of } 200g$
- $10\% = 20g$
- $20 \times 3 = 60g$
- $10\% \text{ of } 630ml = 63ml$
- $630$
- $\frac{3}{5} \text{ of } 75$
- $5 \times 75 = 15$
- $3 \times 15 = 45$
- $5\% \text{ of } 24g$
- $10\% = 2$
- $5\% = 1$
- $\frac{2}{8} \text{ of } 32g$
- $8 \div 32 = 4$
- $4 \times 2 = 8$

## Work scrutiny – Year 2 pupils



- All pupils are working on the same area of mathematics through a problem that allows them to adopt different approaches and depth of thinking and reasoning.
  - Two pupils use the distributive law to find  $12 \times 5p$  by  $10 \times 5 + 2 \times 5$  and show two methods of doubling 60 for calculating the cost of 24 lemons.
  - Another pair express their reasoning well in writing, 'We knew 6 lemons cost 30p so we doubled 30 and the total was 60p. It is double 30p because double six is twelve.'
- Low attainers are supported by suitable practical resources and TA, enabling them to solve the problem. They move from counting in 5s and/or repeated addition to recording multiplication sentences.

# Work scrutiny – teachers A, B, C and D



- Teacher A used cards to help pupils to understand how to rearrange algebraic terms. Teacher D used practical resources to help low attainers engage with the same problem as the rest of the class.
- Teacher B appears to have taught a rule – do the pupils understand why it works?
- Teacher C appears to have taught methods based on 10% for percentages of quantities and on dividing and multiplying for fractions. It is not known whether the conceptual link between fractions and percentages has been made. If not, the teaching has fragmented learning of these topics.
- Overall, the extent to which the four teachers focus on conceptual understanding is variable.

## Work scrutiny – teachers A, B, C and D



- The exercise selected by teacher A is 'intelligent' and the problems encourage thinking and mathematical reasoning. (Teacher A could use some consolidation Qs to support any pupils struggling with Q1-5.)
- Teacher B's word problems may be a mix of one-step and two-step. The pupil's lack of working out and reasoning makes it hard to know what he is thinking. Have pupils been encouraged to set out their working and reasoning?
- Teacher C's problems are routine with no increase in complexity or link made between percentages and fractions.
- All four teachers have set problems within the topics being taught, though the quality varies. How often do pupils solve a mix of problems from across the mathematics curriculum?



# Work scrutiny – your school's books



- Focus on one topic selected from the SoW for each year group under scrutiny. You may wish to choose a strand of mathematics that spans a key stage. For each book, using sticky notes to annotate key points, consider the:
  - teaching approaches, including development of conceptual understanding and reasoning
  - depth and breadth of the work set and tackled, including levels of challenge
  - problem solving.
- Then look across the books for quality and consistency of teaching approach and work set including problems.
- Record findings. How might weaknesses be improved for individual teachers/the school? Any good practice to share?

- Work scrutiny by leaders frequently focuses strongly on teachers' marking and feedback.
- However, marking and assessment policies are often not adapted to capture the most important features of teaching and learning in mathematics.
- Look back at the samples of work of pupils A and B.
- Consider how well the marking by each teacher identifies misconceptions and develops the pupil's understanding.

# Pupil A – teacher's comments

- Teacher's comments identify errors correctly and help the pupil to improve and refine answers.
- Teacher sets a relevant extension question.
- Pupil's response (shaded green) is followed up by teacher, forming a dialogue.

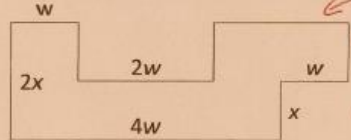
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(+)5x	-4y	-3x	+3y
(+)5x	-3x	-4y	+3y
= 5x	-3x	-4y	+3y
= (5-3)x + (-4+3)y = 2x - y			

- $2x + 3y + 3x + 4y =$   $5x + 7y$  ✓
- $-3u + 5u + 2v - 2v =$   $2u + 0v$  ✓ =  $2u$
- $11x + 3 - 7x - 5 =$   $4x - 2$  ✓
- $p + 3q - 2p - q =$   $-p + 2q$  ✓
- $3m - 2n - m - 3n =$   $2m - 5n$  ✓

6. Find the perimeter of this shape:



*I think you forgot the unmarked sides*

$8w + 3x$  ✗

*Correction please:*

$= 10w + 6x$  ✓

7. Prove that if you add any three consecutive numbers together, the answer is always divisible by 3. [Hint: Let the first number be  $n$ , so the next two are ...]

$n + n + 1 + n + 2 = 3n + 3$

$(3n + 3) \div 3 = n + 1$  ✓ good

*How does this prove it can be divided by 3?*

*I think you've got the idea. ssa 17/10*

*Try this:*  $4(x + y) + 2(x - y) =$

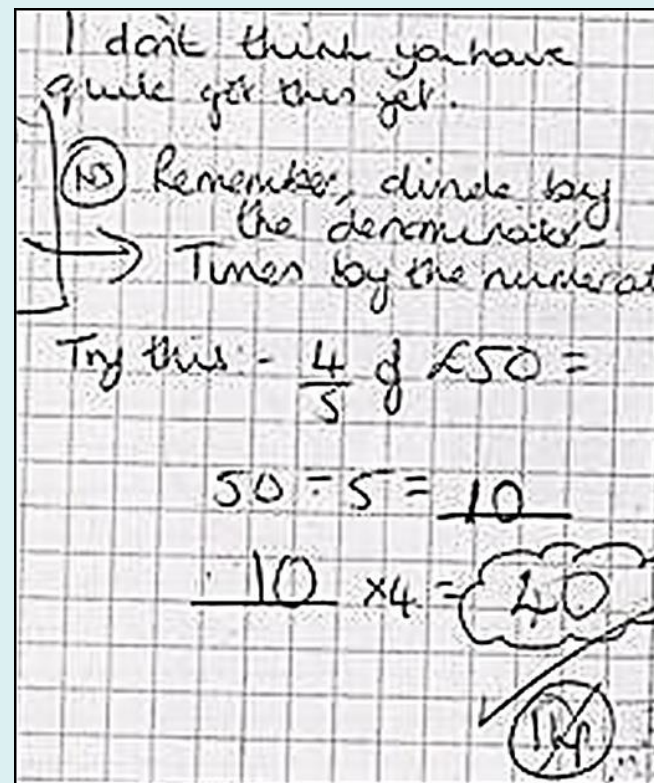
$4x + 4y + 2x - 2y = 6x + 2y$  ✓

*Excellent. ssa 24/10*



## Pupil B – teacher's comments

- In summary, the teacher's comments look helpful at a first glance. They do set up a dialogue with the pupil but they are weak on two counts:
  - They are not pertinent, because the teacher has not recognised the nature of the pupil's errors. The comments imply that the pupil has not applied correctly the method for finding a fraction of a quantity, yet he has worked out correctly all calculations he has written.
  - They emphasise memorising a rote method, without an explanation that helps the pupil to understand why the steps work.



## Good practice in teachers' marking:



- Concentrates on important mathematical aspects, such as misconceptions and recurring errors. Prompts/comments help pupils to see where they have gone wrong, point the way forward, enable pupils to think again and self-correct.
- Includes use of 'what if ...?' and/or 'try this ...' as ways to challenge pupils and/or check they understand.
- Is manageable as well as useful. Careful selection of work set in lessons and for homework can support teachers' better assessment of what pupils understand and can do.
- Might contribute to whole-school literacy through emphasis on mathematical reasoning, correct mathematical presentation and accurate use of mathematical language/symbols.

Think for a moment ...

How well do your work scrutiny records capture important features of teaching and learning in mathematics?



# Work scrutiny records – school W



- With your partner, look at school W's work scrutiny record in your pack.
- The school uses the same form for all subjects. This record was completed by the school's deputy headteacher.
- Identify key strengths and weaknesses in the:
  - design of the form
  - comments recorded on the form.

# Work scrutiny records – school W



## Strengths in design of form:

- the opportunity for the teacher to respond.

## Weaknesses in design of form:

- 'Aspects' are about compliance with superficial features rather than depth of learning. For instance, dating work and marking it regularly do not ensure it is pitched appropriately or understood by the pupils. Learning objectives may be clearly expressed but inappropriate mathematically.

## Strengths in comments:

- DH identifies that some of the next steps (wishes) do not clarify what could be done better.
- DH follows up a previous weakness (in grading attainment).

## Weaknesses in comments:

- No comments are mathematics-specific. They give teacher no feedback on how to improve teaching or learning.
- No reference is made to quality of provision in promoting conceptual understanding or problem solving (eg in the comments on learning objectives and homework).
- Leader does not question expectations. (Many pupils get everything right, as shown by ticks on their work. 3 pupils below expectations, 3 at expected, and 0 above would be low for a typical cohort, especially for the 2 high attainers ).
- Leader does not question whether the SEN pupil's needs are being met well enough.

# Work scrutiny records – school X



- School X's work-scrutiny record summarises findings of a scrutiny of work of pupils taught by three different Key Stage 2 teachers.
- The form and the way it has been completed by the senior leader represent strong practice. Note that each teacher receives individual feedback under the same five headings.
- Look at the record, focusing on the highlighted cells, which contain summaries for each section, each teacher and overall.
- With your partner, identify key strengths in the:
  - design of the school's form
  - comments recorded on the form.

# Work scrutiny records – school X



## Strengths in design of form:

- prompts for each of the key aspects within curriculum, progress, teaching and marking
- summary across classes for each section then overall for each teacher, including follow-up since the last scrutiny and new areas for improvement for teachers and school.



## Strengths in comments:

- Strong emphasis on mathematical detail in each section, synthesised well in summaries. These give areas for development at individual and whole-school/key-stage levels, and include foci for CPD and staff meetings.
- DH understands the important factors in promoting teaching and learning through problem solving, understanding and depth, which inform areas for development.
- Picks up on variation, for example in quality of problem solving, reasoning and textbook use.
- Findings are cross-checked with pupils' progress data, both indicating lack of challenge for high attainers.
- Relevant evaluative comments link well to important aspects.

# Work scrutiny records – school X



## Strengths in comments:

- Summaries capture appropriate areas in which development is needed and are suitable to form the basis of discussions between DH, subject leader and teachers.
- Summaries also provide an appropriate level of detail to combine with evidence from other sources, including achievement data and teaching observation, to inform senior leaders and strategic decisions.
- It is not clear from this document alone how targets are pinpointed or details of support are specified but this information was on individual teachers' feedback sheets.

# Work scrutiny systems in your school



Back at school, think about your work-scrutiny system and look at previously completed records.

- Consider whether your work-scrutiny system is getting to the heart of the matter. How effectively does it evaluate strengths and weaknesses in teaching, learning and assessment, and contribute to improvement in them?
- Identify at least one improvement you can make to each of:
  - your school's work-scrutiny form
  - the way work scrutiny is carried out (e.g. frequency, sample, focus, in/out of lessons, by whom)
  - the quality of evaluation and of development points recorded on the forms
  - the follow-up to work scrutiny.

# Observing teaching



- The characteristics you have been considering today with regard to work scrutiny apply equally well to observations of teaching and learning in mathematics.
- They could also form the basis for discussions with pupils about the mathematics they are learning.
- Observation of teaching can focus on one or more specific features of teaching, learning and assessment, in the same way that you did while scrutinising your pupils' work.
- Evaluations of the effectiveness of teaching should encompass a range of evidence.
- The following slide shows some prompts for key aspects that a school might use to support its records of teaching input and the impact on pupils' learning and progress.

# Prompts for observing teaching



aspect	teacher: input	pupils: impact (individuals & groups)
<i>T &amp; L, assessment</i>	<i>quality of teaching and assessment</i>	<i>mathematical detail of gains in knowledge, skills and understanding</i>
monitoring to enhance progress	observe, question, listen, circulate to check and improve pupils' progress	details of how this increments learning or misses opportunities/fails to enhance it
conceptual understanding	teaching approach: structure, images, reasoning, links	depth of conceptual understanding
mis-conceptions	identify and deal with; design activities that reveal them	detail of misconception and degree to which overcome
practice	intelligent; focus on structure /concept; carefully sequenced (not solely mechanical)	grappling with work to build K, S and/or U; incremental steps/links made; learning from errors
problem solving	real thinking required, for all pupils; not just at end (can use to introduce a concept)	confidence to tackle and persistence; depth of thinking; detail of pupils' chosen methods and mathematics
reasoning, language and symbols	promote written and/or oral reasoning; model, check and correct language/symbols	use of reasoning (oral and written); correct language/symbols; detail of missed or unresolved inaccuracy

# Lesson observation in your school



- Back at school, think about your lesson-observation system and look at previously completed records. Identify improvements you can make to your lesson-observation form to focus on the impact of teaching and assessment on pupils' learning and progress.
- Consider also your lesson-observation system. In particular:
  - the way lesson observation is carried out (e.g. frequency, focus, range of classes and teachers, by whom)
  - the quality of evaluation and of development points recorded on the forms
  - the range of evidence gathered on teaching and assessment and their impact
  - the weight given to progress in evaluating teaching
  - the follow-up to lesson observation.

# The workshop



This morning, we have focused on:

- exploiting activities to improve problem solving and reasoning
- reducing variation in teaching quality by emphasising conceptual understanding
- identifying and overcoming misconceptions
- sharpening the mathematical focus of monitoring, particularly work scrutiny.

Reflect for a moment on how you might work with your colleagues and senior leaders to bring about improvement.

# Your role in leading improvement



We hope this workshop will help you to:

- provide support and challenge for your colleagues
- improve the quality of provision and outcomes in mathematics
- strengthen the insightfulness and impact of your school's mathematics improvement plan.



# Better mathematics conference

Subject leadership workshops – primary

Paul Tomkow HMI

Autumn 2016