Better mathematics conference

Spring 2016

Primary workshops

Information booklet for participants

This booklet contains the information you will need to refer to during the activities.

Questions

Version 1

Version 2

Question 1

Jeans cost £13.95. They are reduced by 1/3 in a sale.
What is their price in the sale?

Dan buys the jeans. He pays with a £10 note.
How much change does he get?

Question 2

Jeans cost £13.95. They are reduced by 1/3 in a sale.

Dan buys the jeans. He pays with a £10 note.
How much change does he get?

Question 3

Jeans cost £13.95. They are reduced by 1/3 in a sale.

Dan has £10. Does he have enough money to buy the jeans?
Explain why.

Question 4

A different pair of jeans is also reduced by 1/3 in a sale.
The sale price is £12.

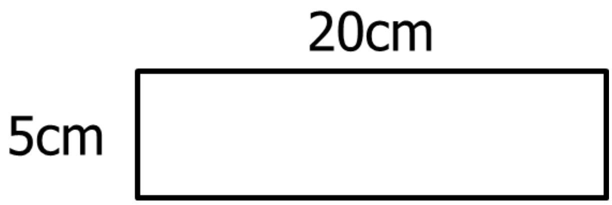
What was the original price?

Deepening a problem

Problems can be adapted by:
- removing intermediate steps
- reversing the problem
- making the problem more open
- asking for all possible solutions
- asking why, so that pupils reason
- asking directly about a mathematical relationship.

Remember, you can:
- improve routine and repetitive questions by adapting them
- set a rich problem or investigation instead
- discuss alternative approaches to solving the problem
- set problems that go more deeply into the topic.

Area of a rectangle



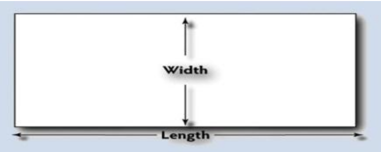
Approaches to other topics

You might find the following general questions useful in discussions with your colleagues about the teaching of other topics, perhaps identified through monitoring of teaching or question level analysis of test results.

1. How well does your introduction develop conceptual understanding?
2. How repetitive are your questions? How soon do you use questions that reflect the breadth and depth of the topic?
3. At what stage do you set problems? How well do they deepen understanding and reasoning?
4. Are questions and problems presented in different ways?

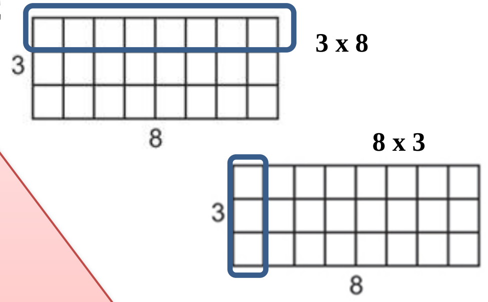
Introduction to the formula for area of a rectangle

Teacher A



I start by telling pupils the formula L x W. They work out lots of areas mentally, when I give them the dimensions or show them a diagram. They write answers on mini-whiteboards. I check they can identify the length and width correctly. Then they do a worksheet with lots of different rectangles; the first few have their lengths written on and the next ones pupils have to measure. The high attainers have questions with larger numbers and decimals.

Teacher C



I ask the pupils to count squares inside the rectangles drawn on the IWB. Then we count systematically. We add rows, for example 8+8+8, which they know is the same as 3 lots of 8. We also add columns, which is 8 lots of 3. The pupils see why the formula works because the dimensions match the numbers of rows and columns. They understand the same area can be built up from rows or columns.

Teaching Assistant B

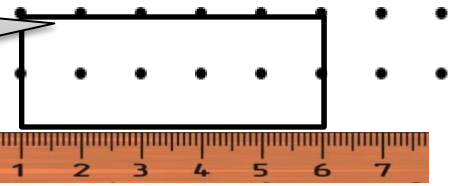
I try to find something that pupils can relate to from their experience – a swimming pool is a good image because most pupils understand what 'a length of the pool' means. They then recognise the 'length' as the longest side.

Teacher D

I give the pupils rectangles on dotty grids so they don't waste time drawing them. I mark the length and width of the first few, and they find the others. I ask pupils to count the squares inside the rectangles, complete the table and look for a connection between the numbers in the table. They sometimes make mistakes but usually spot that multiplying gives the number of squares.

Length
5 or 6?

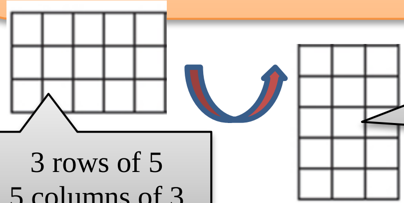
Rectangle D



Rectangle	Length	Width	Squares
A	**4**	**2**	*8*
B	**5**	**3**	*15*
C	**6**	**5**	*29*
D	*6*	*2*	*10*
E	*5*	*4*	*20*

Teacher E

I make a rectangle with 15 tiles and explain its area is 15. We look at it in different orientations using a visualiser. I describe each image in different ways using 'rows' and 'columns'. They understand it is the same rectangle in different positions so has the same area.



3 rows of 5
5 columns of 3

5 rows of 3
3 columns of 5

Then I give each pair 12 tiles and ask them to make a rectangle. I ask if they think they could make another rectangle with 12 tiles, making sure they predict its dimensions before they make it. I mention the word 'factors' if they don't. It's important to ask them to check they've found them all and explain why. Then I give them other numbers of tiles to investigate. In the end, we have discussed factors, primes, the fact that squares are special rectangles and the formula for area of a rectangle being the product of the dimensions.

As well as area, I've learned more about factors, prime numbers and square numbers.
You can only make one rectangle with 11 tiles – and I know why!

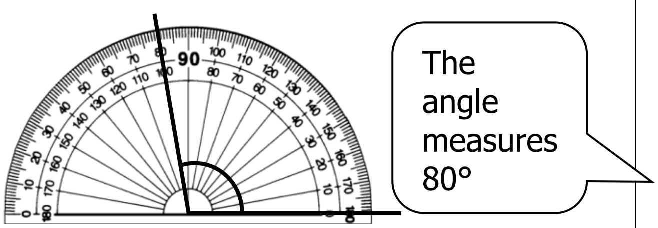
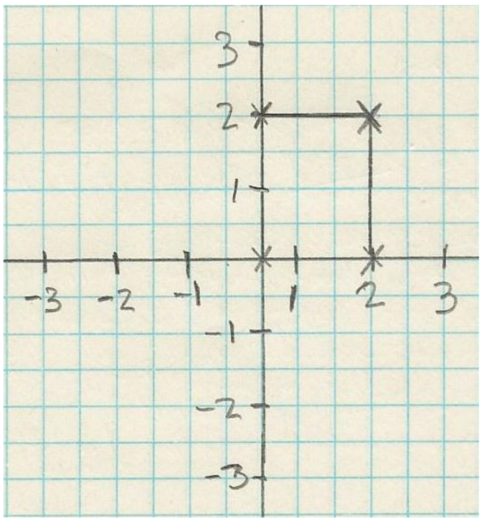
Area	Length	Width
24 cm²	24 cm	1 cm
	12 cm	2 cm
	8 cm	3 cm
	6 cm	4 cm

Misconceptions

Several errors are illustrated below. They are caused by:

- underlying misconceptions
- unhelpful rules
- lack of precision with the order of language and symbols.

With your partner, see if you can identify the underlying misconception or cause of each error.

Take 6 away from 11 $6 - 11 = 5$	$6 \div ½ = 3$
In order, smallest first: 3.2 3.6 3.15 3.82 3.140	$2.7 \times 10 = 2.70$
32.48 = 32.5 to 1dp, *= 33 to the nearest whole number*	*10% of 70 = 70 ÷ 10 = 7* *20% of 60 = 60 ÷ 20 = 3*
	

Ways to help colleagues

Help staff to be aware of misconceptions that:
- pupils may bring to the lesson
- might arise in what is being taught.

Encourage staff, when planning a topic, to discuss mistakes that pupils commonly make in that topic and explore the misconceptions that underpin them. Also help staff to:
- plan lessons to take account of the misconceptions
- look out for misconceptions by circulating in lessons.

Bear in mind, it is more effective to address misconceptions directly than to avoid or describe them. You could give pupils carefully chosen examples to think about deeply. Pupils then have the opportunity to reason for themselves why something must be incorrect.

The potential of work scrutiny

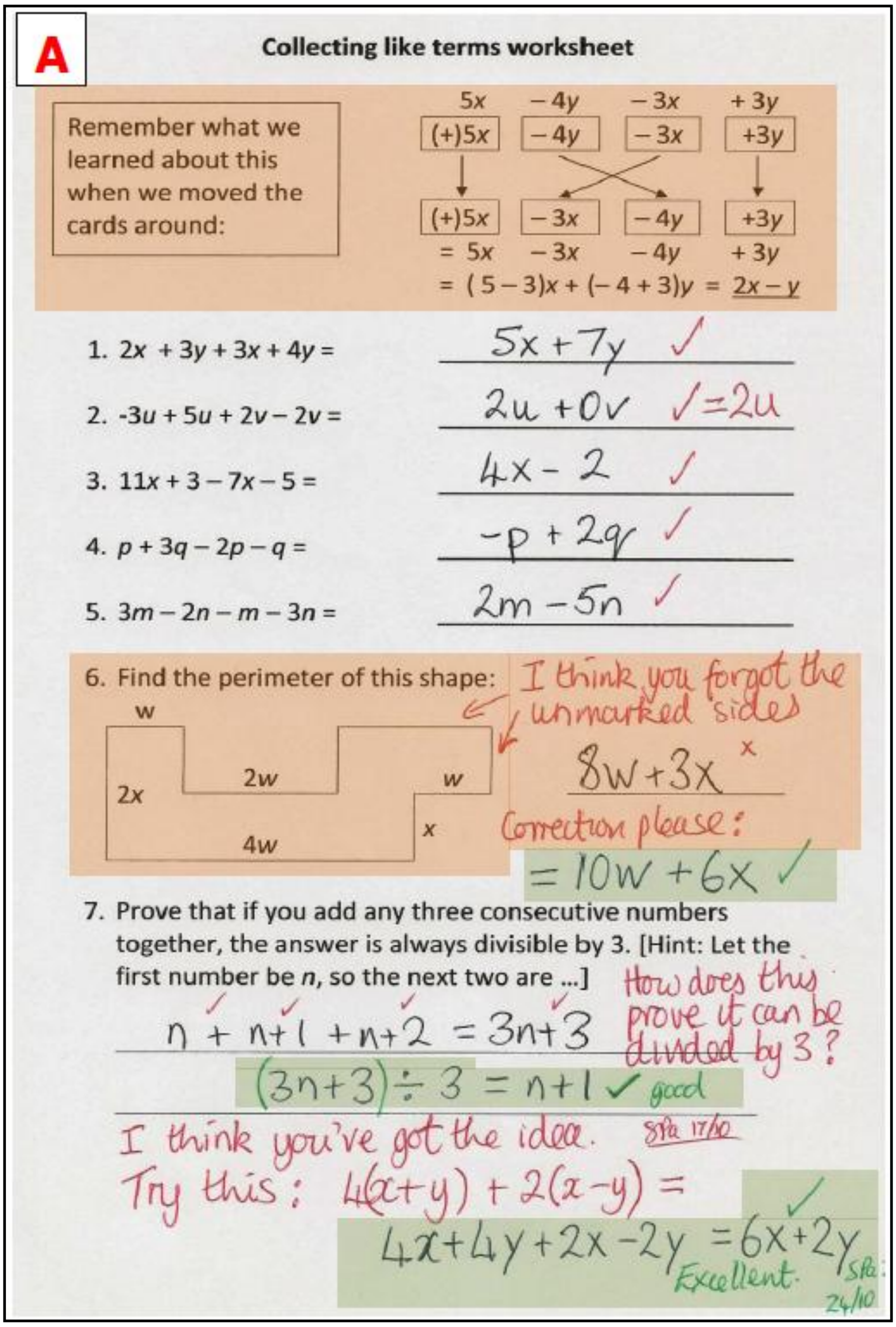
To check and improve:
- teaching approaches, including development of conceptual understanding and reasoning
- depth and breadth of work set and tackled, including levels of challenge
- problem solving
- pupils' understanding and misconceptions
- assessment and its impact on understanding.

To look back over time and across year groups at:
- progression through concepts for pupils of different abilities
- how well pupils have overcome any earlier misconceptions
- balance and depth of coverage of the scheme of work, including reasoning and problem solving.

Extracts of work from pupils in primary classes taught by teachers A, B, C and D

- Look at some/all of the samples of work from pupils in Years 6, 4, 5 and 2. The school has had a focus on increasing problem solving.
- For each piece of work, consider the:
    - teaching approach, including development of conceptual understanding and reasoning
    - depth and breadth of the work set and tackled, including levels of challenge
    - quality of problem solving
- Identify any strengths and weaknesses for each individually and then across the four.

Year 6 – pupil A's work


A

Collecting like terms worksheet

Remember what we learned about this when we moved the cards around:

$5x \quad -4y \quad -3x \quad +3y$

$(+)5x \quad -4y \quad -3x \quad +3y$

$(+)5x \quad -3x \quad -4y \quad +3y$

$= 5x \quad -3x \quad -4y \quad +3y$

$= (5-3)x + (-4+3)y = 2x - y$

1. $2x + 3y + 3x + 4y =$ $5x + 7y$ ✓
2. $-3u + 5u + 2v - 2v =$ $2u + 0v$ ✓ $= 2u$
3. $11x + 3 - 7x - 5 =$ $4x - 2$ ✓
4. $p + 3q - 2p - q =$ $-p + 2q$ ✓
5. $3m - 2n - m - 3n =$ $2m - 5n$ ✓
6. Find the perimeter of this shape: (shape with sides labelled w, 2w, w, 2x, 4w, x)

I think you forgot the unmarked sides

$8w + 3x$ ✗

Correction please:

$= 10w + 6x$ ✓

7. Prove that if you add any three consecutive numbers together, the answer is always divisible by 3. [Hint: Let the first number be *n*, so the next two are ...]

$n + n+1 + n+2 = 3n+3$

How does this prove it can be divided by 3?

$(3n+3) \div 3 = n+1$ ✓ good

I think you've got the idea. SPa 17/10

Try this: $4(x+y) + 2(x-y) =$

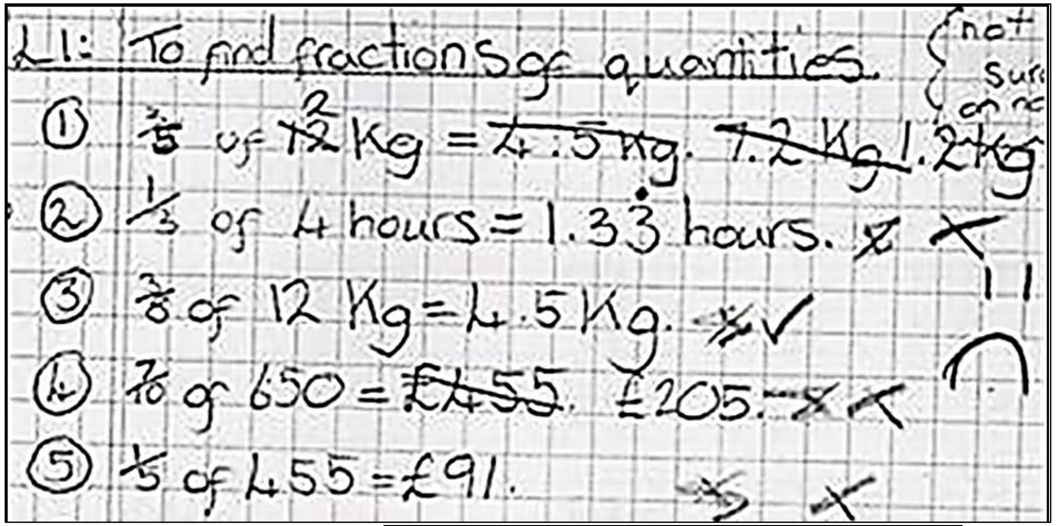
$4x + 4y + 2x - 2y = 6x + 2y$ ✓

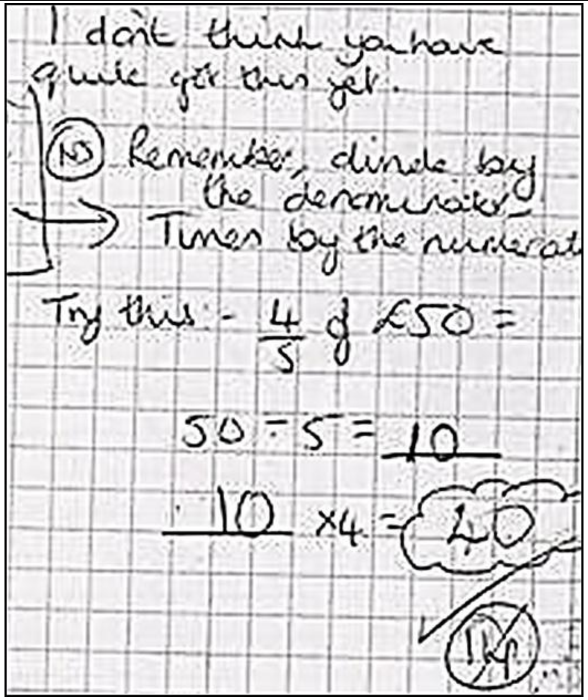
Excellent. SPa 24/10

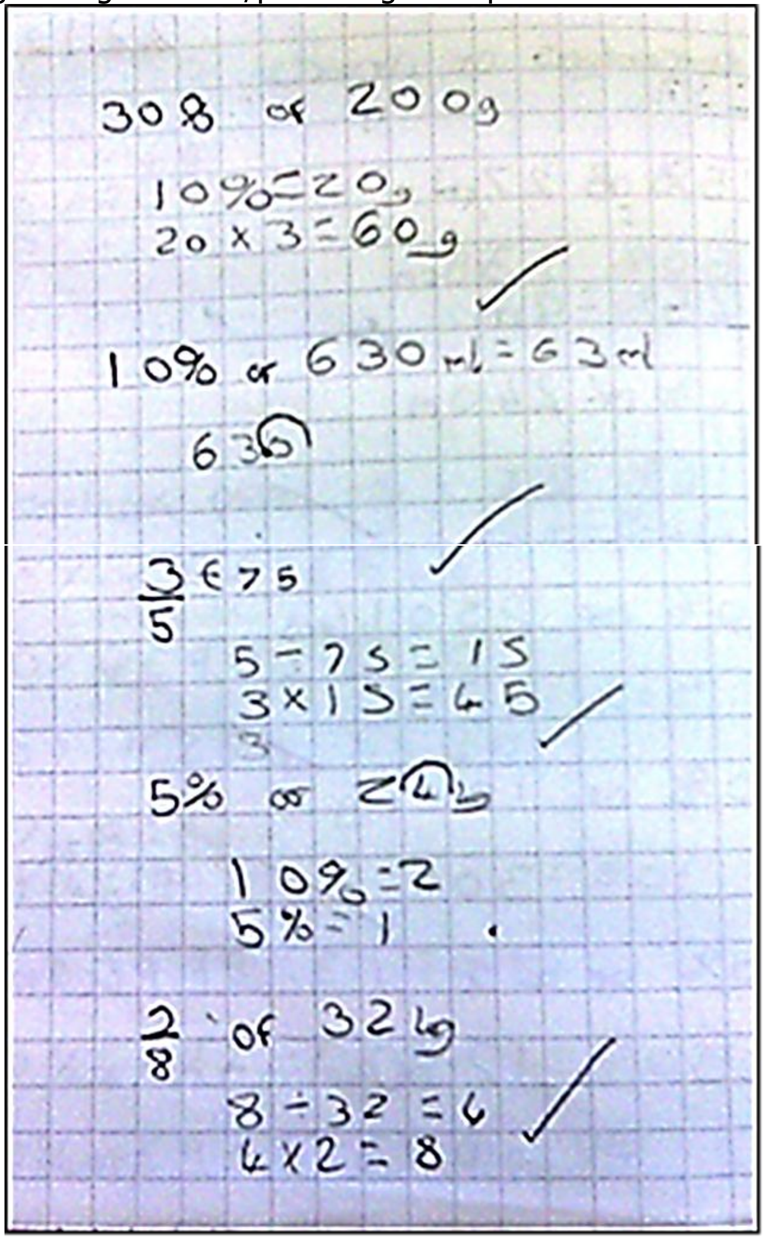
Note: the green highlighting shows pupil A's response to the teacher's comments.

Year 4 – pupil B (pupil's own marking and teacher's comment) **Year 5 – pupil C's work (teacher's marking)**

Pupils B and C have been working on word problems involving finding fractions/percentages of quantities



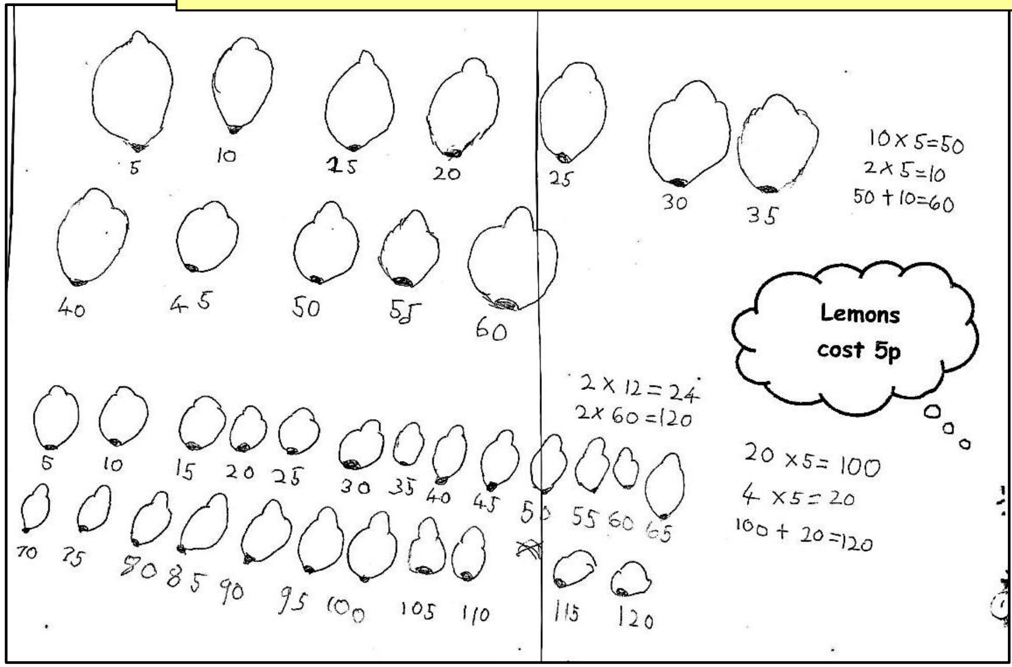


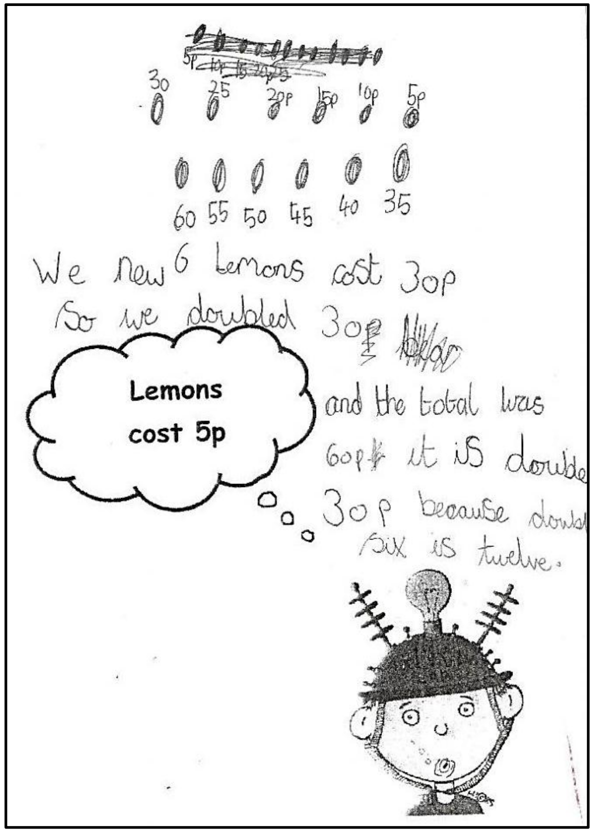


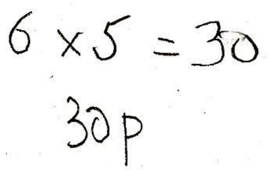
Y2 pupils

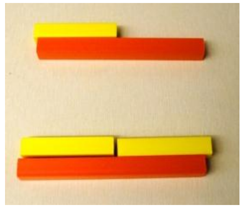
Information about this work
LO: solve problems involving multiplication
Starter: counting up and down in 5s. Recall written form of multiplication facts, eg 4 x 5 = 20
Main: MA and HA on carpet. Worked on cost of 6 lemons at 5p each.
Then in pairs, worked out cost of 12 lemons, then 24 …
Blackbird group worked with TA to build cost of 1-6 lemons using Cuisenaire rods and bead strings and recorded answers on mini-whiteboards.

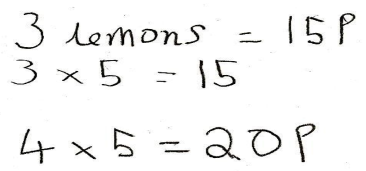
Teacher D











Work scrutiny: your school's books

- Focus on one topic selected from the SoW for each year group under scrutiny. You may wish to choose a strand of mathematics, e.g. fractions, geometric properties, multiplication and division, that spans a key stage.
- For each book, using sticky notes to annotate key points, consider the:
    - teaching approaches, including development of conceptual understanding and reasoning
    - depth and breadth of the work set and tackled, including levels of challenge
    - problem solving.
- Then look across the books for quality and consistency of teaching approach and work set including problems.
- Record findings. How might weaknesses be improved for individual teachers/the school? Any good practice to share?

Marking

- Work scrutiny by leaders frequently focuses strongly on teachers' marking and feedback.
- However, marking and assessment policies are often not adapted to capture the most important features of teaching and learning in mathematics.

- Look back at the samples of work of pupils A and B.
- Consider how well the marking by each teacher identifies misconceptions and develops the pupil's understanding.

Good practice in teachers' marking

Marking that:

- concentrates on important mathematical aspects, such as misconceptions and recurring errors. Prompts/comments help pupils to see where they have gone wrong, point the way forward, enable pupils to think again and self-correct
- includes use of 'what if …?' and/or 'try this …' as ways to challenge pupils and/or check they understand.
- is manageable as well as useful. Careful selection of work set in lessons and for homework can support teachers' better assessment of what pupils understand and can do
- might contribute to whole-school literacy through emphasis on mathematical reasoning, correct mathematical present-ation and accurate use of mathematical language/symbols.

Marking: your school's books

- Look back at the marking of the work you have scrutinised in your pupils' books.
- Consider how well the marking by each teacher identifies misconceptions and errors, and develops, deepens or challenges the pupil's understanding.
- Then look across the books for quality and consistency of marking.
- Record findings. How might weaknesses be improved for individual teachers/the school? Any good practice to share?

Work scrutiny record – school W

Teacher name: *Elspeth Thomas* Date: *6 Nov 2015*

Year group: *5* Subject: *Maths* Sample: *2LA, 2 MA, 2 HA pupils*

Aspect	Comment
Book is neat, with no graffiti	✗ ✓ ✓ *WJ's book has graffiti on – ask pupil* ✓ ✓ ✓ *to cover book*
Work is dated	✓ ✓ ✓ *One missing date spotted by T for PG* ✗ ✓ ✓
Learning objectives are clear	? ✓ ✓ *LO copied out at start of lesson* ✓ ✓ ✓ *WJ's is hard to read*
Work is marked regularly	✓ ✓ ✓ *Ticks on every page – some pupil marking,* ✓ ✓ ✓ *initialled by T or TA*
Marking indicates two stars	✗ ✓ ✓ *WJ: no positives but not much work.* ✓ ✓ ✓ *Others: 'Neat work' 'brilliant' ''really good try', 'You can times HTU by U now'*
Marking includes next steps (a wish)	✓ ✓ ✓ *T writes a next step eg 'factors & multiples',* ✓ ✓ ✓ *but it is not always a wish e.g. 'next step: work faster'.*
Work is assigned an attainment grade (Below, Expected, Above)	*B B B T gave grades on every main piece of marking,* *E E E an improvement since last check*
Effort grades are given	*D A A (latest grades - grades given every week)* *B C A WJ effort often poor; SB one-off, usually A/B*
Evidence of homework	✗ ✓ ✓ *Weekly HW set to practise tables* ✓ ✗ ✓ *WJ not doing HW at all; SB missed latest*

DH's comments: *Elspeth, you are following the marking policy pretty closely. I'm glad to see you are now including attainment grades of B, E, A (below, expected and above). Remember the '2 stars and a wish' rule. Some of your next steps aren't really wishes (what could be done better). NB: I am worried about WJ's book. It is very scruffy and he isn't doing much work in lessons. Can you get him to try a bit harder?*

Teacher's comments: *Thank you for noticing the attainment grades! It's hard to think of a wish sometimes when a pupil gets everything right, but I'll try to work on this.*
As for WJ – he's been very difficult lately – especially since his LSA was off sick. His work was much better when she helped him. I made him stay in at play-time a few times.

Work scrutiny summary record – school X

The scrutiny of KS2 pupils' books was by the deputy head, a mathematics specialist, who made this summary to inform discussions with leaders, including the subject leader. The work included the samples you looked at. Each teacher received individual feedback under the same headings.

<table>
<tr><td colspan="2">Teachers: A (subject leader), B (NQT), C</td><td>Date: 19/11/15</td></tr>
<tr><td colspan="2">Year: 6, 4, 5</td><td>Pupils: 1 LA, 1 MA, 1 HA</td></tr>
<tr><td colspan="3">Curriculum: Links/progression; diff'n & challenge; match to scheme of work; Problem solving/math reasoning</td></tr>
<tr><td>6A: Good links across maths curriculum. Interesting worksheets replace text book in places. Challenge for HA. Work includes problem solving, proof/ reasoning, real-life contexts</td><td>4B: Topics covered in logical order as in scheme. Diffn via different exercises with extension for HA. Problem solving at end of topics but many pupils not recording working or reasoning.</td><td>5C: Appropriate topics but fragmented eg %s using mental approach, fractions of quantities by ÷ and x. No links made. Lack of depth. Limited diffn/challenge for HA. Pedestrian PS.</td></tr>
<tr><td colspan="3">Inconsistency among teachers in diff'n and quality of PS/MR. Varying use of textbook scheme - B using it a lot. A often supplements with interesting work. C curriculum depth is an issue.</td></tr>
<tr><td colspan="3">Progress: Evidence of learning; gains in knowledge, skills and understanding; progress of groups</td></tr>
<tr><td>6A: Pupils learn via mix of routine Qs and Qs that make them think. Good reasoning skills. Well judged when to move pupils of all abilities on to more complex work. Stretch evident for HA</td><td>4B: Pupils often start well but get lost with harder Q/PS. Sometimes B misses the root cause of errors eg MA pupil OK calculating fractions of quantities but not answering what question asks.</td><td>5C: Pupils learn via repetition, but only basic cases. Some misconceptions. Links not being made. HA wasting time on easy work. Not developing deep understanding or skills through PS/MR</td></tr>
<tr><td colspan="3">Tracking data suggests better progress evident in 6A than in other years. HA progress data weaker than MA which is consistent with scrutiny. HA not always stretched enough in 4B,5C. 4B and 5C not developing deep understanding or PS/MR skills. 5C misconceptions evident in work - not overcome?</td></tr>
<tr><td colspan="3">Teaching: Teaching approaches used; focus on understanding, depth, problem solving</td></tr>
<tr><td>6A: Approach for CLT focused on understanding - T knows where pupils likely to go wrong. Good breadth & depth of Qs; some require PS, deep thinking, proof. Uses practical resources; adapts work for LA.</td><td>4B: Increasing emphasis on PS but usually at end of topics. Teaching doesn't always focus on understanding eg rule for fractions of quantities. Pupils not helped to record methods or to reason so difficult to spot misconceptions.</td><td>5C: Some word problems used at end of topics but all same type so little thinking required. Focus on proficient calculation but fragmented, no links or depth. Misconceptions, such as moving the dp, suggest teaching of rules.</td></tr>
<tr><td colspan="3">All setting problems for most topics but vary in quality and in the ways pupils record their methods and reasoning. B (NQT) not consistently spotting what pupils are doing wrong - subject knowledge about common misconceptions? Teaching approaches vary - some use of rules rather than understanding, making links and building progression. Subject leader sets good example - needs to support /inspire others to focus more on understanding. School needs to develop more guidance on teaching approaches, so all have similar expectations re depth, intelligent practice, quality PS and development of MR.</td></tr>
<tr><td colspan="3">Marking: Regularity; identifying misconceptions, strengths, how to improve; follow-up dialogue</td></tr>
<tr><td>6A: Good diagnostic marking spots source of misconceptions/ errors. Also small points eg 2u + 0v = 2u. Good on how to improve, extra challenge & follow-up.</td><td>4B: Mostly ✓× marking. Comments try to help but not getting underneath errors and misconceptions.</td><td>5C: Marking uses ✓ and • for errors (old school policy). Lack of help for pupil to improve. Some comments, tend to be vague or about neatness/ behaviour.</td></tr>
<tr><td colspan="3">Marking all up to date, but variable quality. Weak use of guidance on how to improve. Limited diagnostic marking to root out misconceptions. Only subject leader is marking to good standard.</td></tr>
<tr><td colspan="3">Summary: Imp since last scrutiny, areas for improvement, strengths to share; issues for dept to take forward</td></tr>
<tr><td>6A: Own practice is good, but not influencing other teachers enough yet. Conduct some monitoring jointly with me? Start subject leader course soon.</td><td>4B: NQT so no previous scrutiny Follows guidance on PS but needs support to plan for progression & teach problem solving. Discuss with mentor. Observe Y6, Y2 classes</td><td>5C: Some improvement in PS but bolt-on. Issues around depth and links remain. Must follow new marking policy - check again in three weeks</td></tr>
<tr><td colspan="3"><u>Overall:</u> Some improvement since last year, especially in PS - new subject leader and NQT replacing supply. But, pupil progress and teaching still need to improve. Little effective diff'n. Need to support subject leader to develop team, involve consultant/ school-to-school support? Also talk with NQT's mentor. School CPD needed on teaching approaches that develop understanding, intelligent practice, quality problem solving, reasoning and marking to diagnose misconceptions. Also, staff meeting time to concentrate on diff'n and stretching the more able.</td></tr>
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Work scrutiny systems in your school

Back at school, think about your work-scrutiny system and look at previously completed records.

- Consider whether your work-scrutiny system is getting to the heart of the matter. How effectively does it evaluate strengths and weaknesses in teaching, learning and assessment, and contribute to improvement in them?
- Identify at least one improvement you can make to each of:
  - your school's work-scrutiny form
  - the way work scrutiny is carried out (e.g. frequency, sample, focus, in/out of lessons, by whom)
  - the quality of evaluation and of development points recorded on the forms
  - the follow-up to work scrutiny.

Observing teaching

- The characteristics you have been considering today with regard to work scrutiny apply equally well to observations of teaching and learning in mathematics.
- They could also form the basis for discussions with pupils about the mathematics they are learning.
- Observation of teaching can concentrate on specific features of teaching, learning and assessment in the same way that you focused while scrutinising your pupils' work.
- Evaluations of the effectiveness of teaching should encompass a range of evidence.
- The example below shows some prompts for key aspects that a school might use to support its records of teaching input and the impact on pupils' learning and progress.

aspect	teacher: input	pupils: impact (individuals & groups)
progress	*quality of teaching*	*mathematical detail of gains in understanding, knowledge and skills*
monitoring to enhance progress	observe, question, listen, circulate to check and improve pupils' progress	details of how this increments learning or misses opportunities/fails to enhance it
conceptual understanding	approach: structure, images, reasoning, links	depth of conceptual understanding
problem solving	real thinking required, for all pupils; used to introduce a concept or early on	confidence to tackle and persistence; depth of thinking; detail of pupils' chosen methods and mathematics
misconceptions	identify and deal with; design activities that reveal them	detail of misconception and degree to which overcome
reasoning, language and symbols	model, check and correct	correct reasoning and use of language/symbols; detail of missed or unresolved inaccuracy

Lesson observation in your school

- Back at school, think about your lesson-observation system and look at previously completed records. Identify improvements you can make to your lesson-observation form to focus on the impact of teaching and assessment on pupils' learning and progress.
- Consider also your lesson-observation system. In particular:
  - the way lesson observation is carried out (e.g. frequency, focus, range of classes and teachers, by whom)
  - the quality of evaluation and of development points recorded on the forms
  - the range of evidence gathered on teaching and assessment and their impact
  - the weight given to progress in evaluating teaching
  - the follow-up to lesson observation.

Think for a moment …

How could you support colleagues in deepening problems for the next topic they will be teaching?

How is the formula for area of a rectangle taught in your school?

Identify a topic you would find it helpful to discuss with a group of colleagues in this way:
1. How well does your introduction develop conceptual understanding?
2. How repetitive are the questions you set?
3. At what stage do you set problems? How well do they deepen understanding and reasoning?
4. Are questions and problems presented in different ways?

Take 6 away from 11
6 - 11 = 5
What future learning might be impeded?

How might you find out about misconceptions across the school?

How might you help colleagues use misconceptions well in their teaching?

How well do your work scrutiny records capture important features of teaching and learning in mathematics?

How might you work with colleagues and senior leaders to bring about improvement?
Evaluating progress and teaching